

Lecture No. 40

Measure and Integration

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$f_n \rightarrow f$ in p th mean

To show $f_n \rightarrow f$ in measure

i.e., $\forall \epsilon > 0$

$\mu \{x \in X \mid |f_n(x) - f(x)| \geq \epsilon\} = 0?$

$E = \{x \in X \mid |f_n(x) - f(x)| \geq \epsilon\}$

$f_n \rightarrow f$ in $L_p \equiv \|f_n - f\|_p^p \rightarrow 0$

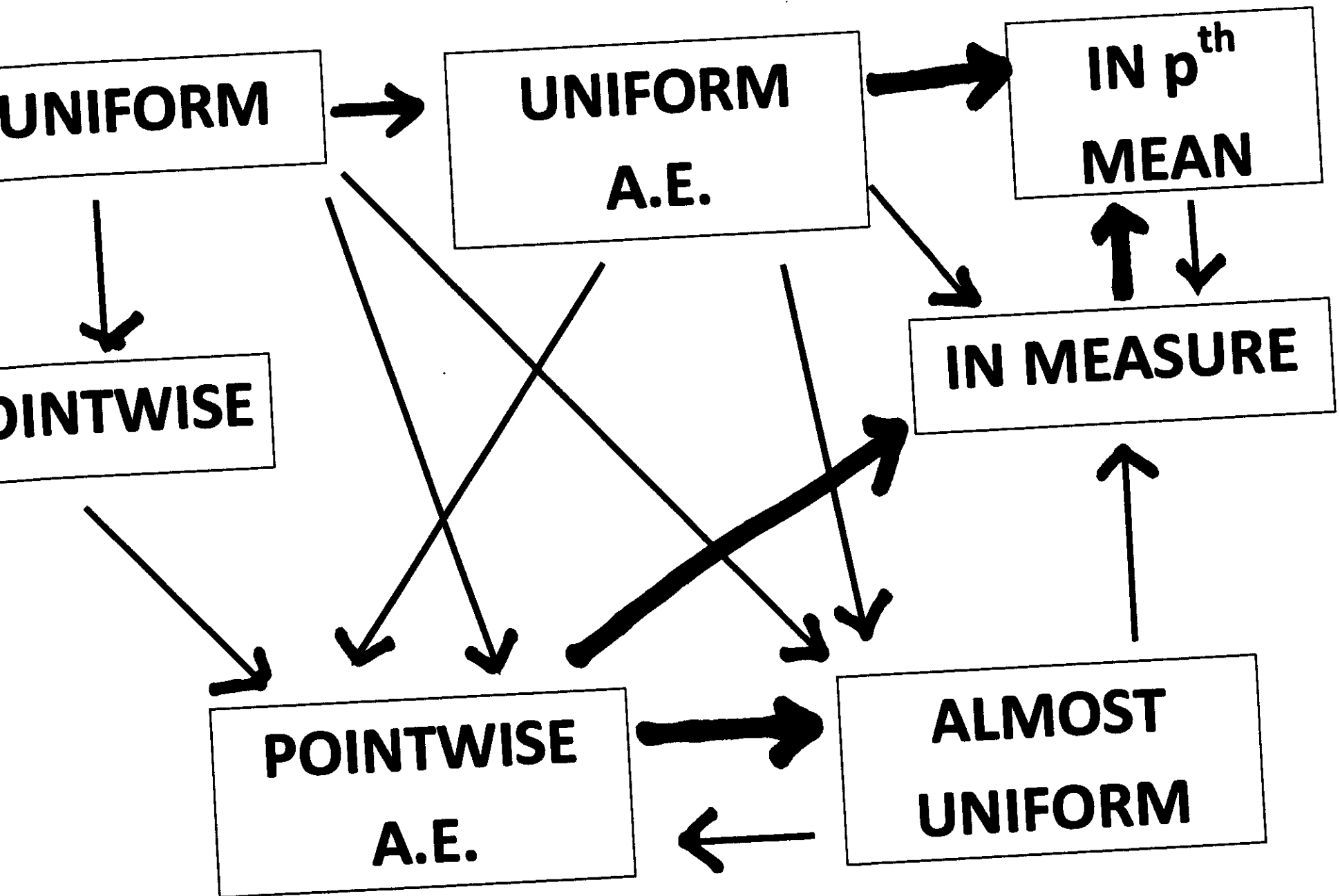
$$\|f_n - f\|_p^p = \int |f_n - f|^p d\mu$$

$$= \int_E |f_n - f|^p d\mu + \int_{E^c} |f_n - f|^p d\mu$$

$$\geq \varepsilon^p \cdot \mu(E)$$

$$\Rightarrow \mu(E) \leq \frac{\|f_n - f\|_p^p}{\varepsilon^p} \rightarrow 0$$

$$\Rightarrow \mu(E) = 0$$



- μ σ -finite
- $\mu(X) < +\infty$